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Interactions of a $j = 1$ Boson in the $2(2j + 1)$ Component Theory

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The amplitudes for boson–boson and fermion–boson interactions are calculated in the second order of perturbation theory in the Lobachevsky space. An essential ingredient of the model is the Weinberg $2(2j + 1)$-component formalism for describing a particle of spin j. The boson–boson amplitude is then compared with the two-fermion amplitude obtained long ago by Skachkov on the basis of the Hamiltonian formulation of quantum field theory on the mass hyperboloid, $p_0^2 - \mathbf{p}^2 = M^2$, proposed by Kadyshevsky. The parametrization of the amplitudes by means of the momentum transfer in the Lobachevsky space leads to the same spin structures in the expressions of T matrices for the fermion and the boson cases. However, certain differences are found. Possible physical applications are discussed.

The scattering amplitude for the two-fermion interaction was obtained in the Lobachevsky space in the second order of perturbation theory as [Skachkov (1975a), equation (31)]

$$T_V^{(2)}(\mathbf{k}(-)\mathbf{p}, \mathbf{p}) = -g_v^2 \frac{4m^2}{\mu^2 + 4æ^2} - 4g_v^2 \frac{(\boldsymbol{\sigma}_1 æ)(\boldsymbol{\sigma}_2 æ) - (\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)æ^2}{\mu^2 + 4æ^2}$$
$$- \frac{8g_v^2 p_0 æ_0}{m^2} \frac{i\boldsymbol{\sigma}_1[\mathbf{p} \times æ] + i\boldsymbol{\sigma}_2[\mathbf{p} \times æ]}{\mu^2 + 4æ^2}$$
$$- \frac{8g_v^2}{m^2} \frac{p_0^2 æ_0^2 + 2p_0 æ_0(\mathbf{p}\cdot æ) - m^4}{\mu^2 + 4æ^2}$$
$$- \frac{8g_v^2}{m^2} \frac{(\boldsymbol{\sigma}_1\mathbf{p})(\boldsymbol{\sigma}_1 æ)(\boldsymbol{\sigma}_2\mathbf{p})(\boldsymbol{\sigma}_2 æ)}{\mu^2 + 4æ^2} \qquad (1)$$

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g_v is the coupling constant. This treatment is based on the use of the formalism of separation of the Wigner rotations and parametrization of currents by means of the Pauli–Lyuban'sky vector (Shirokov, 1951, 1954, 1957, 1958; Chou and Shirokov, 1958; Cheshkov and Shirokov, 1962, 1963; Cheshkov, 1966; Kozhevnikov *et al.*, 1972). The quantities

$$\text{æ}_0 = \left[\frac{m(\Delta_0 + m)}{2}\right]^{1/2}, \qquad \text{æ} = \mathbf{n}_\Delta\left[\frac{m(\Delta_0 - m)}{2}\right]^{1/2}$$

are the components of the 4-vector of a momentum "half-transfer." This concept is closely connected with a notion of the half-velocity of a particle (Chernikov, 1957, 1973). The 4-vector Δ_μ

$$\mathbf{\Delta} = \Lambda_{\mathbf{p}}^{-1}\mathbf{k} = \mathbf{k}(-)\mathbf{p} = \mathbf{k} - \frac{\mathbf{p}}{m}\left(k_0 - \frac{\mathbf{k}\cdot\mathbf{p}}{p_0 + m}\right) \tag{2a}$$

$$\Delta_0 = (\Lambda_p^{-1}k)_0 = (k_0 p_0 - \mathbf{k}\cdot\mathbf{p})/m = (m^2 + \mathbf{\Delta}^2)^{1/2} \tag{2b}$$

can be regarded as the momentum transfer vector in the Lobachevsky space.[3,4]

This amplitude has been used for physical applications in the framework of Kadyshevsky's version of the quasipotential approach (Kadyshevsky, 1968a,b; Kadyshevsky *et al.*, 1972; Skachkov, 1975a,b; Skachkov and Solovtsov, 1978).

[3] I keep the notation and terminology of Skachkov (1975a,b) and Skachkov and Solovtsov (1978). For example, the vector current, taking into account the Pauli term, is given by

$$j^\mu_{\sigma\sigma'}(\mathbf{p},\mathbf{k}) = \bar{u}_\sigma(\mathbf{p})\left\{g_v\gamma^\mu - f_v\frac{\sigma^{\mu\nu}}{2m}q_\nu\right\}u_{\sigma'}(\mathbf{k}), \qquad q = p - k \tag{3}$$

and

$$j^\mu_{\sigma\sigma'}(\mathbf{p},\mathbf{k}) = \sum_{\sigma_p=-1/2}^{1/2} j^\mu_{\sigma\sigma_p}(\mathbf{k}(-)\mathbf{p},\mathbf{p})D^{1/2}_{\sigma_p\sigma'}\{V^{-1}(\Lambda_p,k)\} \tag{4}$$

where $D^{1/2}_{\sigma_p\sigma'}\{V^{-1}(\Lambda_p,k)\} = \xi_{\sigma_p}D^{1/2}\{V^{-1}(\Lambda_p,k)\}\xi_{\sigma'}$; $D^{1/2}(A) \equiv D^{(1/2,0)}(A)$ is the Wigner matrix of the irreducible representation of the $SU(2)$ group (or rotation group). The technique of construction of $D^J(A)$ can be found in Novozhilov (1975, pp. 51, 70).

[4] In general, for each particle in the interaction one should understand under the 4-momenta p_i^μ and k_i^μ ($i = 1, 2$) their covariant generalizations $\mathring{p}_i^\mu$, $\mathring{k}_i^\mu$ (e.g., Shirokov, 1951, 1954, 1957, 1958; Chou and Shirokov, 1958; Cheshkov and Shirokov, 1962, 1963; Cheshkov, 1966; Kozhevnikov *et al.*, 1972; Faustov, 1973; Dvoeglazov *et al.*, 1991):

$$\mathring{\mathbf{k}} = (\Lambda_{\mathcal{P}}^{-1}\mathbf{k}) = \mathbf{k} - \frac{\mathcal{P}}{\sqrt{\mathcal{P}^2}}\left(k_0 - \frac{\mathcal{P}\cdot\mathbf{k}}{\mathcal{P}_0 + \sqrt{\mathcal{P}^2}}\right)$$

$$\mathring{k}_0 = (\Lambda_{\mathcal{P}}^{-1}k)_0 = (m^2 + \mathring{\mathbf{k}}^2)^{1/2}$$

with $\mathcal{P} = p_1 + p_2$, $\Lambda_{\mathcal{P}}^{-1}\mathcal{P} = (\mathcal{M}, \mathbf{0})$. However, we omit the open dots above the momenta in the following, because in the case under consideration we do not miss physical information if we use the corresponding quantities in c.m.s., $\mathbf{p}_1 = -\mathbf{p}_2 = \mathbf{p}$ and $\mathbf{k}_1 = -\mathbf{k}_2 = \mathbf{k}$.

On the other hand, Joos (1962) and Weinberg (1964a,b, 1969) proposed an attractive $2(2j+1)$-component formalism for describing particles of higher spins. As opposed to the Proca 4-vector potentials, which transform according to the (1/2, 1/2) representation of the Lorentz group, the spinor $2(2j+1)$-component functions are constructed via the representation $(j, 0) \oplus (0, j)$ in the Joos–Weinberg formalism. This description of higher spin particles is on an equal footing to the description of the Dirac spinor particle, whose wave function transforms according to the $(1/2, 0) \oplus (0, 1/2)$ representation. The $2(2j+1)$-component analogs of the Dirac functions in the momentum space are[5]

$$\mathcal{U}(\mathbf{p}) = \left(\frac{M}{2}\right)^{1/2} \begin{pmatrix} D^J(\alpha(\mathbf{p}))\xi_\sigma \\ D^J(\alpha^{-1\dagger}(\mathbf{p}))\xi_\sigma \end{pmatrix} \tag{5}$$

for the positive-energy states, and

$$\mathcal{V}(\mathbf{p}) = \left(\frac{M}{2}\right)^{1/2} \begin{pmatrix} D^J(\alpha(\mathbf{p})\Theta_{[1/2]})\xi^*_\sigma \\ D^J(\alpha^{-1\dagger}(\mathbf{p})\Theta_{[1/2]})(-1)^{2J}\xi^*_\sigma \end{pmatrix} \tag{6}$$

for the negative-energy states (Novozhilov, 1975, p. 107), with the following notations:

$$\alpha(\mathbf{p}) = \frac{p_0 + M + (\boldsymbol{\sigma}\cdot\mathbf{p})}{[2M(p_0 + M)]^{1/2}}, \qquad \Theta_{[1/2]} = -i\sigma_2 \tag{7}$$

For instance, in the case of spin $j = 1$, one has

$$D^1(\alpha(\mathbf{p})) = 1 + \frac{(\mathbf{J}\cdot\mathbf{p})}{M} + \frac{(\mathbf{J}\cdot\mathbf{p})^2}{M(p_0 + M)} \tag{8a}$$

$$D^1(\alpha^{-1\dagger}(\mathbf{p})) = 1 - \frac{(\mathbf{J}\cdot\mathbf{p})}{M} + \frac{(\mathbf{J}\cdot\mathbf{p})^2}{M(p_0 + M)} \tag{8b}$$

$$D^1(\alpha(\mathbf{p})\Theta_{[1/2]}) = \left[1 + \frac{(\mathbf{J}\cdot\mathbf{p})}{M} + \frac{(\mathbf{J}\cdot\mathbf{p})^2}{M(p_0 + M)}\right]\Theta_{[1]} \tag{8c}$$

$$D^1(\alpha^{-1\dagger}(\mathbf{p})\Theta_{[1/2]}) = \left[1 - \frac{(\mathbf{J}\cdot\mathbf{p})}{M} + \frac{(\mathbf{J}\cdot\mathbf{p})^2}{M(p_0 + M)}\right]\Theta_{[1]} \tag{8d}$$

($\Theta_{[1/2]}$, $\Theta_{[1]}$ are the Wigner operators for spin 1/2 and 1, respectively). In spite of the relative antiquity of this formalism, in our opinion it does not deserve to be retired. From the phenomenological viewpoint this approach provides the necessary framework for constructing a QCD-based effective

[5] These functions obey the orthonormalization equation, $\mathcal{U}^\dagger(\mathbf{p})\gamma_{00}\mathcal{U}(\mathbf{p}) = M$, where M is the mass of the Joos–Weinberg particle. A similar normalization condition exists for $\mathcal{V}(\mathbf{p})$, the function of "negative-energy states."

field theory of higher spin hadronic resonances and could yield new insights into the quark structure of these excited hadrons. Recently, much attention has been paid to this formalism (Ahluwalia and Ernst, 1992a,b, 1993; Ahluwalia *et al.*, 1993a; Ahluwalia and Goldman, 1993) [see also Sankaranarayanan and Good (1965a,b), Sankaranarayanan (1965), and Dvoeglazov (1994a–d) regarding similar problems]. Unfortunately, the older work devoted to this formalism missed the possibility of another definition of negative-energy bispinors $\mathcal{V}(\mathbf{p}) = S^c_{[1]}\mathcal{U}(\mathbf{p}) \equiv \mathcal{C}_{[1]}\mathcal{K}\mathcal{U}(\mathbf{p}) \sim \gamma_5\mathcal{U}(\mathbf{p})$, like the Dirac $j = 1/2$ case.[6] Here $S^c_{[1]}$ is the charge conjugation operator for $j = 1$ (Ahluwalia *et al.*, 1993a); $\mathcal{K}$ is the operation of complex conjugation. This definition, based on the use of another form of the Ryder–Burgard relation (Ahluwalia *et al.*, 1993a; Ahluwalia and Goldman, 1993), leads to a different physical content: in the latter case a boson and its antiboson have opposite relative intrinsic parities (like Dirac spinor particles). This is an example of another class of Poincaré-invariant theories (Bargmann–Wightman–Wigner-type quantum field theories (Wigner, 1965). This remarkable fact, which has been proven in Ahluwalia *et al.* (1993a) and Ahluwalia and Goldman (1993), hints that the problem of the adequate choice of the field operator has profound physical significance.

The Feynman diagram technique has been discussed (Weinberg, 1964a,b, 1969; Hammer *et al.*, 1968; Tucker and Hammer, 1971; Shay and Good, 1969; Novozhilov, 1975; Dvoeglazov and Skachkov, 1984, 1987, 1988) in the above six-component formalism for particles of spin $j = 1$, using the Lagrangian[7,8]

$$\mathcal{L} = \overline{\Psi}(x)\Gamma_{\mu\nu}\overleftarrow{\nabla}_\mu\overrightarrow{\nabla}_\nu\Psi(x) - M^2\overline{\Psi}(x)\Psi(x) - \frac{1}{4}F_{\mu\nu}F_{\mu\nu}$$

$$+ \frac{e\lambda}{12}F_{\mu\nu}\overline{\Psi}(x)\gamma_{5,\mu\nu}\Psi(x) + \frac{e\kappa}{12M^2}\partial_\alpha F_{\mu\nu}\overline{\Psi}(x)\gamma_{6,\mu\nu,\alpha\beta}\nabla_\beta\Psi(x) \quad (9)$$

In the above formula we have $\overleftrightarrow{\nabla}_\mu = -i\partial_\mu \mp eA_\mu$; $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor; A_μ is the 4-vector of the electromagnetic field; $\overline{\Psi}$, Ψ are the six-component wave functions (WF) of a massive $j = 1$ Joos–Weinberg particle. The following expression has been obtained for the interaction vertex of the particle with the vector potential (Hammer *et al.*,

[6] I do not treat here the new Majorana-like constructs in the $(j, 0) \oplus (0, j)$ representation space (Ahluwalia *et al.*, 1993b, 1994a,b), referring the reader to our recent work (Dvoeglazov, 1994e, 1995).

[7] In the following I prefer to use the Euclidean metric because this metric has been applied in many papers on the $2(2j + 1)$ formalism.

[8] The expression for the Lagrangian has been generalized in Dvoeglazov (1994a–d). In this paper we are still going to use the previous one in order to emphasize features of the formalism relevant to our purposes.

1968; Tucker and Hammer, 1971; Shay and Good, 1969; Dvoeglazov and Skachkov, 1984, 1987)

$$-e\Gamma_{\alpha\beta}(p + k)_\beta - \frac{ie\lambda}{6}\gamma_{5,\alpha\beta}q_\beta + \frac{e\kappa}{6M^2}\gamma_{6,\alpha\beta,\mu\nu}q_\beta q_\mu(p + k)_\nu \quad (10)$$

where $\Gamma_{\alpha\beta} = \gamma_{\alpha\beta} + \delta_{\alpha\beta}$, $\gamma_{\alpha\beta}$, $\gamma_{5,\alpha\beta}$, $\gamma_{6,\alpha\beta,\mu\nu}$ are the $6 \otimes 6$ matrices which have been described in Barut *et al.* (1963) and Weinberg (1964a,b, 1969)

$$\gamma_{ij} = \begin{pmatrix} 0 & \delta_{ij}1 - J_iJ_j - J_jJ_i \\ \delta_{ij}1 - J_iJ_j - J_jJ_i & 0 \end{pmatrix} \quad (11a)$$

$$\gamma_{i4} = \gamma_{4i} = \begin{pmatrix} 0 & iJ_i \\ -iJ_i & 0 \end{pmatrix}, \qquad \gamma_{44} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (11b)$$

and

$$\gamma_{5,\alpha\beta} = i[\gamma_{\alpha\mu}, \gamma_{\beta\mu}]_- \quad (12a)$$

$$\gamma_{6,\alpha\beta,\mu\nu} = [\gamma_{\alpha\mu}, \gamma_{\beta\nu}]_+ + 2\delta_{\alpha\mu}\delta_{\beta\nu} - [\gamma_{\beta\mu}, \gamma_{\alpha\nu}]_+ - 2\delta_{\beta\mu}\delta_{\alpha\nu} \quad (12b)$$

J_i are the spin matrices for a $j = 1$ particle, e is the electron charge, and λ and κ are quantities that correspond to the magnetic dipole moment and the electric quadrupole moment, respectively.

In order to obtain the 4-vector current for the interaction of a Joos–Weinberg boson with the external field one can use the formulas of Skachkov (1975a,b), Skachkov and Solovtsov (1978), Shirokov (1951, 1954, 1957, 1958), Chou and Shirokov (1958), Cheshkov and Shirokov (1962, 1963), Cheshkov (1966), and Kozhevnikov *et al.* (1972), which are valid for any spin:

$$\mathcal{U}^\sigma(\mathbf{p}) = S_\mathbf{p}\mathcal{U}^\sigma(\mathbf{0}), \qquad S_\mathbf{p}^{-1}S_\mathbf{k} = S_{\mathbf{k}(-)\mathbf{p}}\cdot I \otimes D^1\{V^{-1}(\Lambda_p, k)\} \quad (13)$$

$$W_\mu(\mathbf{p})\cdot D\{V^{-1}(\Lambda_p, k)\} = D\{V^{-1}(\Lambda_p, k)\}\cdot\left[W_\mu(\mathbf{k}) + \frac{p_\mu + k_\mu}{M(\Delta_0 + M)}p_\nu W_\nu(\mathbf{k})\right] \quad (14)$$

$$k_\mu W_\mu(\mathbf{p})\cdot D\{V^{-1}(\Lambda_p, k)\} = -D\{V^{-1}(\Lambda_p, k)\}\cdot p_\mu W_\mu(\mathbf{k}) \quad (15)$$

W_μ is the Pauli–Lyuban'sky 4-vector of relativistic spin. The matrix $D^J\{V^{-1}(\Lambda_p, k)\}$ is written for spin $j = 1$ as follows:

$$\begin{aligned} &D^{(j=1)}\{V^{-1}(\Lambda_p, k)\} \\ &= \frac{1}{2M(p_0 + M)(k_0 + M)(\Delta_0 + M)}([\mathbf{p} \times \mathbf{k}]^2 \\ &+ [(p_0 + M)(k_0 + M) - \mathbf{k}\cdot\mathbf{p}]^2 + 2i[(p_0 + M)(k_0 + M) \\ &- \mathbf{k}\cdot\mathbf{p}]\{\mathbf{J}\cdot[\mathbf{p} \times \mathbf{k}]\} - 2\{\mathbf{J}\cdot[\mathbf{p} \times \mathbf{k}]\}^2) \end{aligned} \quad (16)$$

However, the formulas obtained in Dvoeglazov and Skachkov (1988)[9]

$$S_{\mathbf{p}}^{-1}\gamma_{\mu\nu}S_{\mathbf{p}} = \gamma_{44}\left\{\delta_{\mu\nu} - \frac{1}{M^2}\chi_{[\mu\nu]}(\mathbf{p}) \otimes \gamma_5 - \frac{2}{M^2}\Sigma_{[\mu\nu]}(\mathbf{p})\right\} \tag{17a}$$

$$S_{\mathbf{p}}^{-1}\gamma_{5,\mu\nu}S_{\mathbf{p}} = 6i\left\{-\frac{1}{M^2}\chi_{(\mu\nu)}(\mathbf{p}) \otimes \gamma_5 + \frac{2}{M^2}\Sigma_{(\mu\nu)}(\mathbf{p})\right\} \tag{17b}$$

where

$$\chi_{[\mu\nu]}(\mathbf{p}) = p_\mu W_\nu(\mathbf{p}) + p_\nu W_\mu(\mathbf{p}) \tag{18a}$$

$$\chi_{(\mu\nu)}(\mathbf{p}) = p_\mu W_\nu(\mathbf{p}) - p_\nu W_\mu(\mathbf{p}) \tag{18b}$$

$$\Sigma_{[\mu\nu]}(\mathbf{p}) = \frac{1}{2}\{W_\mu(\mathbf{p})W_\nu(\mathbf{p}) + W_\nu(\mathbf{p})W_\mu(\mathbf{p})\} \tag{18c}$$

$$\Sigma_{(\mu\nu)}(\mathbf{p}) = \frac{1}{2}\{W_\mu(\mathbf{p})W_\nu(\mathbf{p}) - W_\nu(\mathbf{p})W_\mu(\mathbf{p})\} \tag{18d}$$

lead to the 4-current of a $j = 1$ Joos–Weinberg particle more directly[10]:

[9] Attention is drawn to the definition of the γ_5 matrix, which differs by a sign from the definition used in Shay and Good (1969) and Dvoeglazov and Skachkov (1988).

[10] Compare with the $j = 1/2$ case:

$$S_{\mathbf{p}}^{-1}\gamma_\mu S_{\mathbf{p}} = \frac{1}{m}\gamma_0\{1 \otimes p_\mu + 2\gamma_5 \otimes W_\mu(\mathbf{p})\} \tag{19a}$$

$$S_{\mathbf{p}}^{-1}\sigma_{\mu\nu}S_{\mathbf{p}} = -\frac{4}{m^2}1 \otimes \Sigma_{(\mu\nu)}(\mathbf{p}) + \frac{2}{m^2}\gamma_5 \otimes \chi_{(\mu\nu)}(\mathbf{p}) \tag{19b}$$

and then

$$j_\mu^{\sigma_p\nu_p}(\mathbf{k}(-)\mathbf{p}, \mathbf{p}) = \frac{1}{m}\xi_{\sigma_p}^{\dagger}\{2g_v æ_0 p_\mu + f_v æ_0 q_\mu + 4g_{\mathcal{M}}W_\mu(\mathbf{p})(\boldsymbol{\sigma}\cdot\mathbf{æ})\}\xi_{\nu_p}, \qquad g_{\mathcal{M}} = g_v + f_v \tag{20}$$

The indices $\mathbf{p}$ indicate that the Wigner rotations have been separated out and thus all spin indices have been "reset" on the momentum $\mathbf{p}$. One can rewrite (Skachkov, 1975b) the electromagnetic current (20):

$$j_\mu^{\sigma_p\nu_p}(\mathbf{k}, \mathbf{p}) = -\frac{em}{æ_0}\xi_{\sigma_p}^{\dagger}\left\{g_{\mathcal{E}}(q^2)(p+k)^\mu + g_{\mathcal{M}}(q^2)\left[\frac{1}{m}W_\mu(\mathbf{p})(\boldsymbol{\sigma}\cdot\boldsymbol{\Delta}) - \frac{1}{m}(\boldsymbol{\sigma}\cdot\boldsymbol{\Delta})W_\mu(\mathbf{p})\right]\right\}\xi_{\nu_p} \tag{21}$$

$g_{\mathcal{E}}$ and $g_{\mathcal{M}}$ are the analogs of the Sachs electric and magnetic form factors. Thus, if we regard $g_{S,T,V}$ as effective coupling constants depending on the momentum transfer, we can assure ourselves that the form of the currents for a spinor particle and for a $j = 1$ boson is the same (with Wigner rotations separated out).

$$j_{\mu}^{\sigma_p\nu_p}(\mathbf{p}, \mathbf{k}) = j_{\mu(S)}^{\sigma_p\nu_p}(\mathbf{p}, \mathbf{k}) + j_{\mu(V)}^{\sigma_p\nu_p}(\mathbf{p}, \mathbf{k}) + j_{\mu(T)}^{\sigma_p\nu_p}(\mathbf{p}, \mathbf{k}) \tag{22a}$$

$$j_{\mu(S)}^{\sigma_p\nu_p}(\mathbf{p}, \mathbf{k}) = -g_S\xi_{\sigma_p}^{\dagger}\left\{(p + k)_\mu\left(1 + \frac{(\mathbf{J}\cdot\mathbf{\Delta})^2}{M(\Delta_0 + M)}\right)\right\}\xi_{\nu_p} \tag{22b}$$

$$j_{\mu(V)}^{\sigma_p\nu_p}(\mathbf{p}, \mathbf{k}) = -g_V\xi_{\sigma_p}^{\dagger}\left\{(p + k)_\mu + \frac{1}{M} W_\mu(\mathbf{p})(\mathbf{J}\cdot\mathbf{\Delta}) - \frac{1}{M}(\mathbf{J}\cdot\mathbf{\Delta})W_\mu(\mathbf{p})\right\}\xi_{\nu_p} \tag{22c}$$

$$j_{\mu(T)}^{\sigma_p\nu_p}(\mathbf{p}, \mathbf{k}) = -g_T\xi_{\sigma_p}^{\dagger}\left\{-(p + k)_\mu \frac{(\mathbf{J}\cdot\mathbf{\Delta})^2}{M(\Delta_0 + M)} + \frac{1}{M} W_\mu(\mathbf{p})(\mathbf{J}\cdot\mathbf{\Delta}) - \frac{1}{M}(\mathbf{J}\cdot\mathbf{\Delta})W_\mu(\mathbf{p})\right\}\xi_{\nu_p} \tag{22d}$$

Let us note an interesting feature (Ahluwalia and Ernst, 1993; Dvoeglazov, 1994c). The 6-spinors $\mathcal{U}(\mathbf{p})$ and $\mathcal{V}(\mathbf{p})$ defined by (5) and (6) do not form a complete set:

$$\frac{1}{M}\{\mathcal{U}(\mathbf{p})\overline{\mathcal{U}}(\mathbf{p}) + \mathcal{V}(\mathbf{p})\overline{\mathcal{V}}(\mathbf{p})\} = \begin{pmatrix} 1 & S_{\mathbf{p}} \otimes S_{\mathbf{p}} \\ S_{\mathbf{p}}^{-1} \otimes S_{\mathbf{p}}^{-1} & 1 \end{pmatrix} \tag{23}$$

But if we take $\mathcal{V}_2(\mathbf{p}) = \gamma_5\mathcal{V}(\mathbf{p})$, we can obtain the complete set. Fortunately,

$$\overline{\mathcal{V}}_2(\mathbf{0})\mathcal{U}_1(\mathbf{0}) = 0 \tag{24}$$

what permits us to keep the parametrization (4). As a matter of fact, in (22b)–(22d) we have used the second definition of negative-energy spinors (Ahluwalia *et al.*, 1993a; Ahluwalia and Goldman, 1993).

Next, let me represent the Feynman matrix element corresponding to the diagram of the two-boson interaction, mediated by the particle described by the vector potential, in the form (Skachkov, 1975a,b; Skachkov and Solovtsov, 1978; Dvoeglazov and Skachkov, 1984, 1987) (see footnote 4)

$$\begin{aligned}&\langle p_1, p_2; \sigma_1, \sigma_2|\hat{T}^{(2)}|k_1, k_2; \nu_1, \nu_2\rangle\\ &= \sum_{\sigma_{ip},\nu_{ip},\nu_{ik}=-1}^{1} D_{\sigma_1\sigma_{1p}}^{\dagger\,(j=1)}\{V^{-1}(\Lambda_{\mathcal{P}}, p_1)\}D_{\sigma_2\sigma_{2p}}^{\dagger\,(j=1)}\{V^{-1}(\Lambda_{\mathcal{P}}, p_2)\}\\ &\times T_{\sigma_{1p}\sigma_{2p}}^{\nu_{1p}\nu_{2p}}(\mathbf{k}(-)\mathbf{p}, \mathbf{p})D_{\nu_{1p}\nu_{1k}}^{(j=1)}\{V^{-1}(\Lambda_{p_1}, k_1)\}D)_{\nu_{1k}\nu_1}^{(j=1)}\{V^{-1}(\Lambda_{\mathcal{P}}, k_1)\}\\ &\times D_{\nu_{2p}\nu_{2k}}^{(j=1)}\{V^{-1}(\Lambda_{p_2}, k_2)\}D_{\nu_{2k}\nu_2}^{(j=1)}\{V^{-1}(\Lambda_{\mathcal{P}}, k_2)\}\end{aligned} \tag{25}$$

where

$$T_{\sigma_{1p}\sigma_{2p}}^{\nu_{1p}\nu_{2p}}(\mathbf{k}(-)\mathbf{p}, \mathbf{p}) = \xi_{\sigma_{1p}}^{\dagger}\xi_{\sigma_{2p}}^{\dagger}T^{(2)}(\mathbf{k}(-)\mathbf{p}, \mathbf{p})\xi_{\nu_{1p}}\xi_{\nu_{2p}} \tag{26}$$

$\xi^\dagger$, ξ are the analogs of Pauli spinors. The calculation of the amplitude (26) yields ($p_0 = -ip_4$, $\Delta_0 = -i\Delta_4$)

$$\hat{T}^{(2)}(\mathbf{k}(-)\mathbf{p}, \mathbf{p})$$
$$= g^2\left\{\frac{[p_0(\Delta_0 + M) + (\mathbf{p}\cdot\mathbf{\Delta})]^2 - M^3(\Delta_0 + M)}{M^3(\Delta_0 - M)}\right.$$
$$+ \frac{i(\mathbf{J}_1 + \mathbf{J}_2)\cdot[\mathbf{p}\times\mathbf{\Delta}]}{\Delta_0 - M}\left[\frac{p_0(\Delta_0 + M) + \mathbf{p}\cdot\mathbf{\Delta}}{M^3}\right]$$
$$+ \frac{(\mathbf{J}_1\cdot\mathbf{\Delta})(\mathbf{J}_2\cdot\mathbf{\Delta}) - (\mathbf{J}_1\cdot\mathbf{J}_2)\mathbf{\Delta}^2}{2M(\Delta_0 - M)}$$
$$\left. - \frac{1}{M^3}\frac{\mathbf{J}_1\cdot[\mathbf{p}\times\mathbf{\Delta}]\mathbf{J}_2\cdot[\mathbf{p}\times\mathbf{\Delta}]}{\Delta_0 - M}\right\} \qquad (27)$$

We have assumed $g_S = g_V = g_T$ above, motivated by group-theoretic reasons and by the analogy discussed in footnote 10. Expression (27) reveals the advantages of the $2(2j + 1)$ formalism, since it looks like the amplitude for the interaction of two spinor particles with the substitutions

$$\frac{1}{2M(\Delta_0 - M)} \Rightarrow \frac{1}{\mathbf{\Delta}^2} \qquad \text{and} \qquad \mathbf{J} \Rightarrow \boldsymbol{\sigma}$$

The calculations hint that many analytical results for a Dirac fermion could be applicable to describing a $2(2j + 1)$ particle. Nevertheless, an adequate explanation is required of the obtained difference. Very likely, its origin lies at the kinematical level. Free-space (without interaction) Joos–Weinberg equations admit acausal tachyonic solutions (Ahluwalia and Ernst, 1992b): "Interaction introduced in the massive Weinberg equations will couple to both the causal and acausal solutions and thus cannot give physically reasonable results." However, we have used the Tucker and Hammer (1971) approach, which does not possess tachyonic solutions.[11]

For the sake of completeness I also present the amplitudes for interaction of $j = 1$ and $j = 0$ particles, $j = 1/2$ and $j = 0$ particles, and $j = 1/2$ and $j = 1$ particles. We use the equation for the 4-current of a spinor particle ($f_\nu = 0$) defined by formula (21), equation (22b) with $\mathbf{J} = 0$ for a scalar particle (e.g., Rohrlich, 1950), and equation (22c) for the 4-current of a $j = 1$ particle in the Joos–Weinberg formalism. Following the technique of "resetting" polarization indices, we obtain in the first case

[11] I am not going to deal further with this subject in the present paper. The description of dynamics based on a new kinematical basis (Ahluwalia and Ernst, 1992a, 1993; Ahluwalia *et al.*, 1993a; Ahluwalia and Goldman, 1993) will be given in detail elsewhere.

$$\hat{T}^{(2)}_{01}(\mathbf{k}, \mathbf{p}) = -\frac{g_0 g_1}{2m_1(\Delta_1^0 - m_1)} \{-2m_1^2(\Delta_1^0 + m_1)$$
$$+ [p_1^0(\Delta_1^0 + m_1) + (\mathbf{p}\cdot\boldsymbol{\Delta}_1)](p_1^0 + p_2^0 + k_1^0 + k_2^0)$$
$$+ i\mathbf{J}\cdot[\mathbf{p} \times \boldsymbol{\Delta}_1](p_1^0 + p_2^0 + k_1^0 + k_2^0)\} \tag{28}$$

which has a similar form to $\hat{T}^{(2)}_{0(1/2)}(\mathbf{k}, \mathbf{p})$, which is given below.

As a result of lengthy calculations one can write the boson–fermion amplitudes in the following form:

$$\hat{T}^{(2)}_{0(1/2)}(\mathbf{k}, \mathbf{p}) = -\frac{2g_0 g_{1/2}}{(2m_1)^{3/2}(\Delta_1^0 - m_1)(\Delta_1^0 + m_1)^{1/2}} \{-2m_1^2(\Delta_1^0 + m_1)$$
$$+ [p_1^0(\Delta_1^0 + m_1) + (\mathbf{p}\cdot\boldsymbol{\Delta}_1)](p_1^0 + p_2^0 + k_1^0 + k_2^0)$$
$$+ i\boldsymbol{\sigma}\cdot[\mathbf{p} \times \boldsymbol{\Delta}_1](p_1^0 + p_2^0 + k_1^0 + k_2^0)\} \tag{29}$$

and

$$\hat{T}^{(2)}_{1(1/2)}(\mathbf{p}, \mathbf{k}) =$$
$$-\frac{2g_1 g_{1/2}}{(2m_1)^{3/2}(\Delta_1^0 - m_1)(\Delta_1^0 + m_2)^{1/2}} \Bigg\{-2m_1^2(\Delta_1^0 + m_1)$$
$$+ [p_1^0(\Delta_1^0 + m_1) + (\mathbf{p}\cdot\boldsymbol{\Delta}_1)](p_1^0 + p_2^0 + k_1^0 + k_2^0)$$
$$+ i\boldsymbol{\sigma}_1\cdot[\mathbf{p} \times \boldsymbol{\Delta}_1](p_1^0 + p_2^0 + k_1^0 + k_2^0)$$
$$- \frac{i}{m_2}\mathbf{J}_2\cdot[\mathbf{p} \times \boldsymbol{\Delta}_2]\left[(p_1^0 + p_2^0)(\Delta_1^0 + m_1) + \frac{(\mathbf{p}\cdot\boldsymbol{\Delta}_1)(p_1^0 + p_2^0 + m_1 + m_2)^2}{2(p_1^0 + m_1)(p_2^0 + m_2)}\right]$$
$$- m_1\{(\boldsymbol{\sigma}_1\cdot\boldsymbol{\Delta}_2)(\mathbf{J}_2\cdot\boldsymbol{\Delta}_1) - (\boldsymbol{\sigma}_1\cdot\mathbf{J}_2)(\boldsymbol{\Delta}_1\cdot\boldsymbol{\Delta}_2) + i\mathbf{J}_2\cdot[\boldsymbol{\Delta}_1 \times \boldsymbol{\Delta}_2]\}$$
$$+ \boldsymbol{\sigma}_1\cdot[\mathbf{p} \times \boldsymbol{\Delta}_1]\, \mathbf{J}_2\cdot[\mathbf{p} \times \boldsymbol{\Delta}_2]\, \frac{(p_1^0 + p_2^0 + m_1 + m_2)^2}{2m_2(p_1^0 + m_1)(p_2^0 + m_2)}\Bigg\} \tag{30}$$

We have used the notation

$$\boldsymbol{\Delta}_1 = \mathbf{k} - \frac{\mathbf{p}}{m_1}\left(k_1^0 - \frac{\mathbf{k}\cdot\mathbf{p}}{p_1^0 + m_1}\right), \qquad \Delta_1^0 = (\boldsymbol{\Delta}_1^2 + m_1^2)^{1/2} \tag{31a}$$

$$\boldsymbol{\Delta}_2 = \mathbf{k} - \frac{\mathbf{p}}{m_2}\left(k_2^0 - \frac{\mathbf{k}\cdot\mathbf{p}}{p_2^0 + m_2}\right), \qquad \Delta_2^0 = (\boldsymbol{\Delta}_2^2 + m_2^2)^{1/2} \tag{31b}$$

and $p_1^0 = (\mathbf{p}^2 + m_1^2)^{1/2}$, $k_1^0 = (\mathbf{k}^2 + m_1^2)^{1/2}$, $p_2^0 = (\mathbf{p}^2 + m_2^2)^{1/2}$, $k_1^0 = (\mathbf{k}^2 + m_2^2)^{1/2}$.

DISCUSSION AND POSSIBLE PHYSICAL APPLICATIONS

The main result of this paper is the boson–boson amplitude calculated in the framework of the Joos–Weinberg theory. The separation of the Wigner rotations permits us to reveal certain similarities with the $j = 1/2$ case. Thus, the result provides a basis for the following conclusion: if the existence of the Joos–Weinberg bosons could be proven,[12] many calculations produced earlier for fermion–fermion interactions mediated by the vector potential could be applicable to processes involving this matter structure. Moreover, the main result of this paper gives a certain hope of the possibility of a unified description of fermions and bosons. Now, I realize that all the above mentioned is not surprising. The principal features of describing the particle world on the basis of relativistic quantum field theory are not in some special representation of the group, $(1/2, 0) \oplus (0, 1/2)$ or $(1, 0) \oplus (0, 1)$, or $(1/2, 1/2)$, but in the Lorentz group itself. Several older papers (e.g., Ohmura, 1956) and recent work (Bruce, 1995) support this conclusion. However, the difference between the denominators of the amplitudes necessitates further study of the $(1/2, 0) \oplus (0, 1/2)$ and $(1, 0) \oplus (0, 1)$ representations. These representations, of course, are contained in the general scheme of Joos and Weinberg.

After the appearance of Ahluwalia and Ernst (1992b) and Dvoeglazov (1994a–d) (see also Evans, 1994) we seem to be forced to use equations of this approach[13]

$$[\gamma^{\mu_1\mu_2\cdots\mu_{2j}}\partial_{\mu_1}\partial_{\mu_2}\cdots\partial_{\mu_{2j}} + M^{2j}]\Psi(x) = 0 \tag{32}$$

for describing higher spin particles. In the framework of the standard Proca/Rarita–Schwinger approach we deal with many contradictions in the particle interpretation of the field transforming on the corresponding representations of the Lorentz group: e.g., acausal solutions even at the kinematical level; the absence of a well-defined massless limit; nonconsistency after the introduction of interactions; longitudity of the antisymmetric tensor field after quantization, which contradicts the classical limit and the Weinberg theorem; longitudinal nonpropagating solutions of Maxwell's equations, which could lead to speculations on a "massive" photon, "tired" light (Evans, 1994) and the loss of renormalizability of modern quantum field models, etc.

Second, it was realized that a gluon can be described as a massive particle with dynamical mass appearing due to the existence of the color

[12] As mentioned in this paper and in Ahluwalia *et al.* (1993a) and Ahluwalia and Goldman (1993), probably the crucial experiment for the Joos–Weinberg boson could be based on the determination of relative intrinsic parities of a boson and its antiboson.

[13] See Ahluwalia *et al.* (1993a) and Ahluwalia and Goldman (1993) for a discussion of the possible additional term $\not{p}_{\mu,\nu}$ in the mass term for integer spins.

charge and the self-interaction. This treatment permits one to eliminate some contradictions in the results of calculations of the proton form factor and the effective coupling constant $\alpha_S(q^2)$ on the basis of QCD [for recent discussion see Field (1994) and Consoli and Field (1994)]. Therefore, the presented amplitude could serve as a basis for describing gluonium, the bound state of two massive gluons. The fermion–boson amplitudes could be applied for describing the quark–diquark composite system.

For 30 years the quasipotential approach to quantum field theory (Logunov and Tavkhelidze, 1963; Kadyshevsky, 1968a,b; Kadyshevsky *et al.*, 1972) was regarded to be the most convenient and sufficiently general formalism for calculation of energy spectra of composite states. In the Bethe–Salpeter approach one has a nonphysical parameter (relative time), difficulties with the normalization of the bound-state wave function, etc. All this necessitates constraints on the wave function. As a matter of fact, they lead to approaches which are equivalent to the quasipotential one. For recent discussion see Crater *et al.* (1992) and Dvoeglazov *et al.* (1993b, 1994). Therefore, one can use equations for the equal-time (quasipotential) wave function to achieve the goals discussed above. For example, for a composite system formed by a fermion and a boson of nonequal mass the following equation was given (in the Kadyshevsky version of the quasipotential approach) in Linkevich *et al.* (1983):

$$2\mathring{p}_2^0(\mathcal{M} - \mathring{p}_1^0 - \mathring{p}_2^0)\Phi_{\sigma_1\sigma_2}(\mathring{\mathbf{p}}) = \frac{1}{(2\pi)^3}\sum_{\nu_1\nu_2}\int \frac{d^3\mathring{\mathbf{k}}}{2\mathring{k}_1^0} V_{\sigma_1\sigma_2}^{\nu_1\nu_2}(\mathring{\mathbf{p}}, \mathring{\mathbf{k}})\Phi_{\nu_1\nu_2}(\mathring{\mathbf{k}}) \tag{33}$$

For work dealing with the phenomenological description of hadrons in the $(j, 0) \oplus (0, j)$ framework see Dvoeglazov and Khudyakov (1993, 1994), Dvoeglazov (1994f), and Dvoeglazov *et al.* (1993a).

Finally, not having any intentions to criticize theories based on the concept of vector potential, in our opinion, the principal question is not yet solved. It is not in formal advantages of one or another formalism for describing $j = 1$ (or higher spin) particles, but in "the nature of Nature's mesons."

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